How fuel prices determine public transport infrastructure, modal shares and urban form

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1. Introduction

Urban form has been identified as a crucial dimension of sustainable cities (Glaeser and Kahn, 2010; Weisz and Steinberger, 2010). Population density is a simple metric of urban form. While population density itself remains insufficient to explain a plethora of interactions between inhabitants

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and the built environment, it nonetheless relates to two crucial dimensions of urban transport and energy conversation. First, higher population density tends to be associated with shorter distances to travel, reducing energy demand. In a global comparison of cities, urban population density is inversely correlated with urban transport energy use (Newman and Kenworthy, 1989). This correlation is weaker in US cities once controlled for accessibility of destinations and street network design (Ewing and Cervero, 2010), and is subject to more specific metrics of urban form (Mindali et al., 2004). Second, higher density enables financially viable public transit, which usually is more energy efficient than individual motorized transport (Bongardt et al., 2013). Public transit activity is negatively correlated with private transport activity (Newman and Kenworthy, 1996). A minimum density is seen as a prerequisite for financially viable and environmentally effective public transport (Frank and Pivo, 1994; Cervero, 1998; Bongardt et al., 2010).

While causalities remain somewhat unclear in this literature, there is a disparate discipline - urban economics - which has mostly developed around models explaining the interaction between transport and urban form. The model framework dates back to von Thünen in the early 19th century, who explored the relationship between agricultural product choice, land rent, and transport distance to the central market place (von Thünen, 1826). It was Alonso, and later Muth and Mills, who transferred this framework to residential location choice, and commuting costs to the central business district (CBD) (Alonso, 1964; Fujita, 1989). In their model, increasing commuting costs are compensated by decreased land rent, resulting in decreasing population density towards the urban fringe. In turn, lower marginal transport costs imply more urban sprawl.

This world of models from urban economics remains surprisingly unconnected to the empirical data and correlation studies initiated by Newman and Kenworthy (1989). This paper connects an urban economic framework to questions of population density, mode choice, relying both on analytical and numerical observations. The focus of this paper is on the optimal municipal investment decision, which optimizes total utility when public transit and population density influence each other. We use this model to shed light on possible causal relationships between population density, modal share and energy consumption, reengaging with the empirical database of Newman and Kenworthy (1989), Newman and Kenworthy (1996). We believe that an engaged discussion between urban economists and urban transport planers is fruitful for both communities.

In Section 2, we introduce the model, in which public transit is fully endogenous to urban form and generalized costs of private transport. In this model, generalized transport costs, urban form and mode choice are nonlinearly related to each other. In Section 3, we present numerical results demonstrating the interrelationship between transport costs, population density and modal share. We also demonstrate that free-market provision of public transit is socially inefficient. Finally, in Section 4 we show that this model can reproduce some relevant results from Newman and Kenworthy (1989), Newman and Kenworthy, 1996, in particular the relationships between population density and transport energy use, and between transport distance of different modes.

### 2. A bi-modal city

This section introduces a density and modal share modeling framework – based on the Alonso–Mills–Muth model of a monocentric city with transport costs and housing market. The model builds on a substantial literature base from urban economics.

Mills suggested to expand the AMM model to additional transport modes, a recommendation taken up by some of his peers. Capoza (1973) emphasized the tradeoff between capital and land as scarce input factors for subway versus road infrastructure: in the inner city, land becomes so scarce and valuable that land-intensive road infrastructure is substituted by capital-intensive subway lines (Capoza, 1973). Haring et al. (1976) demonstrate that an additional transport mode reduces the land rent differential between CBD and urban fringe (Haring et al., 1976). Anas and Moses (1979) not only specify two modes but also explore the role of discrete transport corridors, producing a number of varying urban forms as a function of generalized costs in dense and sparse radial transport networks (Anas and Moses, 1979). Also in 1979, Kim, a PhD student of Mills, published a two-mode model in which a city with two million inhabitants has sufficient population density to support a subway system,
in contrast to a city with only one million inhabitants (Kim, 1979). Borck and Wrede (2008) and Proost and Van Dender (2008) have made progress in addressing optimal mode choice in presence of income heterogeneities and externalities (Borck and Wrede, 2008; Proost and Van Dender, 2008). Sasaki (1989) studied the impact of a change in fixed and variable costs on rent levels, welfare and spatial structures in an urban system with two modes. Our model is in mode specification comparable to that of Sasaki (1989), but including an additional infrastructure component. Su and DeSalvo (2008) confirms empirically that urban area contracts with public transit subsidies but expands with an auto subsidy. Crucially, Brueckner (2005) demonstrates that transport subsidies are warranted if there are increasing returns to scales in one mode as might be the case for rail-based public transit (Brueckner, 2005). From a political economy perspective Brueckner and Selod (2006) point to an underinvestment in infrastructure of the expensive transportation mode (car) if voters have heterogeneous skills (Brueckner and Selod, 2006). However, and somewhat surprisingly, the endogeneity of marginal public transport costs to urban form and marginal car costs has not yet been studied. We try to fill this gap here and make the results relevant to the empirical studies of global cities.

\[ F_c: \text{ Fixed costs of car ownership} \]
\[ m_c: \text{ Marginal costs of car driving (} m_c). \text{ Includes fuel and insurance costs, as well as time costs, and is, hence, also dependent on the quality and scope of road infrastructure. In DEMOS, } m_c \text{ is the key variable driving urban form and modal shares} \]
\[ m_p: \text{ Marginal price of public transit charged to patrons per unit distance. In the social planner equilibrium } m_p \equiv m_{infra}; \text{ in the market solution, } m_p = F_c/r_n + m_c. \text{ Note that } m_p(r) \to \infty \text{ for } r > r_p (r_p \text{ is defined below}). \]
\[ m_{infra}: \text{ Marginal costs of public transit per unit distance due to infrastructure provision. In the social planner equilibrium } m_{infra} \equiv m_p; \text{ in the market solution, } m_{infra} \leq m_p \]
\[ C: \text{ Infrastructure cost of public transit per unit area} \]
\[ z: \text{ Composite consumer good} \]
\[ s: \text{ Lot size of residence} \]
\[ Y: \text{ Aggregate income} \]
\[ T(r): \text{ Transport costs of commuter living at } r \]
\[ r_p: \text{ Radius of the city area in which public transit is used} \]
\[ r_n: \text{ Radius of the city area covered by public transit infrastructure} \]
\[ r_c: \text{ Radius of the total city area as served by cars} \]
\[ R(r): \text{ Unit rent costs of commuter living at } r \]
\[ R_a: \text{ Agricultural land rent at } r_c \]
\[ \rho(r): \text{ Population density at } r \]
\[ T: \text{ Average total transport costs of commuters} \]
\[ R: \text{ Average rents costs of commuters} \]

2.1. Household location in a monocentric city

We introduce the well-known model framework – a household location theory – following Alonso (1964) who generalizes the agricultural bid rent theory of von Thünen (1826). We specify a closed-city model (the population is constant) and public land ownership (in contrast to land lord ownership of the land) (Fujita, 1989). The city is characterized as monocentric with a dense radial transport system without congestion. All travel consists of commuters who travel from their residences to the city center where their work is located. The land is featureless, and public goods and externalities are absent.

**Economic agent 1: The Households.** Households maximize utility \( U(z, s) \) where \( z \) is a composite consumer good, and \( s \) is the lot size of the house. Furthermore, rents are denoted as \( R(r) \), and transport costs as \( T(r) \), assumed to be monotonically increasing in \( r \). The optimization problem of each household is given by

\[
\max_{z, s} U(z, s), \quad \text{with} \quad z = Y - T(r) - sR(r).
\]
For identical individuals with identical income and utility function, households locate in equilibrium such that utility is equal, i.e. \( U(z, s) = u \) being constant in equilibrium. If otherwise, some households could improve their utility by moving. The household income is derived from its occupation in an external non-urban economy \( Y_0 \), land rent income \( R \), and – in the case of the market solution – from the profit of the public transit provider \( P \) (see Fig. 1).

**Economic agent 2: The Public Landowners.** Land belongs to the public and the average rent is part of the overall income (see Fig. 1). The land rent \( R(r) \) is maximized at any given distance. No housing construction market is specified here. Hence, land rent \( R(r) \) is only a function of land demand and not of housing stock. In this model, the market land rent always coincides with the bid rent in equilibrium (Fujita, 1989).

For given utility \( U(z, s) = u \), the log-linear transform of a Cobb-Douglas utility function is given as
\[
U(z, s) = \alpha \log z + \beta \log s
\]
with \( \alpha + \beta = 1 \), and \( \alpha > 0 \), and \( \beta > 0 \). Under these conditions, it can be shown that rent costs and lot size are given as (Fujita, 1989):
\[
R(r, u) = \frac{\alpha}{\beta} \frac{Y - T(r)}{e^{-u/\beta}},
\]
\[
s(r, u) = \frac{\alpha}{\beta} \frac{Y - T(r)}{e^{-u/\beta}}.
\]
(1)

The density profile, or urban form, is then given as \( \rho(r, u) = \frac{1}{2\pi r s(r, u)} \). Furthermore, in a radial symmetric city the city area is specified as a function of distance to the city center as \( A(r) = 2\pi r dr \). The city boundary is denoted as \( r_c \), corresponding to the outer radius of the car transportation mode (see below).

In this model, utility \( U(z, s) \) changes with different transport costs and density profile. In contrast, total population \( N \) is kept constant. According to the classification of, e.g., (Fujita, 1989), fixed population and variable utility corresponds to a closed city model. This allows to solve utility as a function of population and transport costs.

\[
N = \int_0^{r_c} \frac{2\pi r}{s(r, u)} dr.
\]
(2)

---

**Fig. 1.** Model framework. The partial equilibrium framework of urban economics here chosen relies on income derived from an external production sector, recycles rent income and – in the case of the market solution – the profit from the public transit sector. Car producers, oil companies and the construction sector are outside of the urban economy (internalizing the construction sector would further accentuate the results).
Substituting Eq. 1 into Eq. 2 and solving for $u$, we obtain utility directly as a function of transport costs:

$$U(z, s) = \alpha \log z + \beta \log s$$

$$T_p(r) = F_c + m_p r$$

$$T_e(r) = F_e + m_e r$$

$$R(r, u) = \alpha^{\alpha/\beta} \beta (Y - T(r))^{1/\beta} e^{-u/\beta}$$

$$S(r, u) = \alpha^{-\alpha/\beta} (Y - T(r))^{-1/\beta} e^{u/\beta}.$$

$c$: consumption of composite good

$z$: consumption of land

$T$: transport costs

$R$: unit rent costs

$Y$: income

**Fig. 2.** Structure of a common transport land-rent model with two modes.

Substituting Eq. 1 into Eq. 2 and solving for $u$, we obtain utility directly as a function of transport costs:

$$u(r, T(r)) = -\beta \ln N \left[ \int_0^{r_p} \frac{2\pi r}{2^{2/\beta} (Y - T(r))^{2/\beta}} dr \right]^{-1}.$$ (3)

We can normalize with respect to the population, i.e., without loss of generality, $N = 1$. In the following, we assume the equilibrium land use market of the CCA model as given and investigate social welfare exclusively for optimal public transport provision.\(^1\)

### 2.2. Endogenized costs of public transport

So far, transport $T(r)$ is a function assumed to be increasing in $r$. We now specify transport costs and introduce two modes, public transport and cars. Each mode is characterized by marginal user costs and fixed capital costs. The marginal costs are denoted as $m_p$ for public transport and $m_c$ for car transport. Public transport is assumed to have no capital costs for users, whereas car users need to invest into vehicles, $F_c > F_p = 0$. Transport costs $T_i(r)$ are then given as follows: (See Fig. 2)

$$T_p(r) = m_p r$$

$$T_e(r) = F_c + m_e r.$$ (4) (5)

In an inner circle $r < r_p$, public transport is more economic than car transport. More precisely, mode choice is given as follows:

$$i = p: \quad 0 < r < r_p$$

$$i = c: \quad r_p < r < r_e.$$

\(^1\) As an interesting background note: Is the closed-city model with land-rent optimization socially optimal? If one uses a Benthamite welfare function and adds individual utilities, the model is generally not optimal. In fact, the discretionary nature of land allocation and asymmetry caused by transport costs produces asymmetrical production possibilities: total welfare can more easily be enhanced by giving additional land to those living farther from the city. As a result, inequality is socially optimal in this model. This observation has been first characterized by Mirrlees (1972), and been analytically treated by Arnott and Riley (1977). As a correlate, in the absence of externalities (summarized, e.g., in Brueckner (2000)) and with Cobb-Douglas utility function, social welfare is optimized by subsidizing marginal transport costs (Gusdorf and Hallegatte, 2007). In a Herbert-Stevens model the social surplus for given identical utility level is maximized. Under such conditions, it is always possible to find an income tax such that the market equilibrium is also efficient (Fujita, 1989).
Here, \( r_p \) denotes the outer radius of the public transport area and \( r_c \) denotes the outer radius of car usage, i.e. \( R(r_c) = R_a \). So for the model implications are straightforward and causally monodirectional (see Fig. 2).

But in addition to this model structure, and as a central property of our model, we aim to endogenize public transit as a function of the population density profile and \( m_c \) (see Fig. 3). More precisely, marginal costs of public transit depend on ridership. Ridership itself depends on the proportion of the population living close to the city center for whom public transit is more attractive than car driving, i.e. on urban form - which itself is a function of the marginal costs of both modes of transportation.

**Economic agent 3: The Municipal Government.** To operationalize this interdependence, consider that public transport relies on infrastructure (e.g. a capital intensive subway system), to be financed by a municipal government, which builds the infrastructure conditional to a budget constraint, but is not profit maximizing. The automobile mode, of course, depends also on the provision of a road infrastructure. Here we postulate that the unit cost of road infrastructure are much smaller than those of public transit and, for ease of exposition, set them to zero.

The area unit transit infrastructure costs are denoted as \( C \). Infrastructure costs are recovered via marginal pricing of public transport. Crucially, this pricing is a function of urban form. Varying marginal costs of car driving (\( m_c \), e.g., fuel pricing including fuel taxes) influence urban form and, hence, the viability of public transport. Overall, the marginal operation costs per passenger and per distance, are given by total service provision per overall ridership.

Let us start with the second component, overall ridership. To calculate this, we need to determine some other variables. The total proportion of people living at distance \( r \) is given by

\[
    n(r) = 2\pi r \rho(r).
\]

(Remember that the total population is normalized, i.e. \( N = 1 \)). The total distance traveled is then

\[
    D(r) = \int_0^r n(r) r dr = \int_0^r 2\pi r^2 \rho(r) dr = \int_0^r 2\pi r^2 \frac{\rho}{S(r)} dr = \int_0^r 2\pi r^2 x^{2/\beta} e^{-\gamma x} e^{-u/\beta} dr. \quad (6)
\]

Now, we can turn our attention to the first term, infrastructure costs. The unit area costs of public transport infrastructure are denoted by \( C \). The municipal government chooses to provide public transport infrastructure within radius \( r_n \). The total area covered by public transport is \( \pi r_n^2 \). Hence, the total provision costs of public transport within \( r_n \) are denoted by \( C\pi r_n^2 \). Then the average costs from supply side are

\[
    m_{\text{infra}} = \frac{C\pi r_n^2}{D(r_n, \rho(m_c, r_n))}.
\]
Or, in other terms, the public transit provider’s budget condition is
\[ m_{\text{infra}} D(r_n, \rho(m_c, r_n)) = C \pi r_n^2. \]  
(7)

The public transport infrastructure term essentially makes the average costs of public transport a function of the other variables, in particular of the marginal costs of car transport \( m_c \) via urban form \( \rho(r) \).

For completeness, let us also define the average transport costs \( T \) and average rent costs \( R \) of a city with radius \( r_c \):
\[ T = \int_0^{r_c} n(r) T(r) \, dr, \]
\[ R = \int_0^{r_c} n(r) R(r) \, dr. \]

\( R \) is part of the overall income of household (see Fig. 1).

3. Providing public transport infrastructure

The model focusses on the relationship between transport costs, density and urban form. In the following we present and visualize the model results.

3.1. Optimal provision of public transport

We impose the no-profit public transit budget condition \( m_{\text{infra}}(r_n) = m_p \) as a side constraint, implying that the supply side costs of public transport equal demand side costs. This condition immediately implies that \( r_p = r_n \). To see that consider that \( r_p > r_n \) is physically prohibited (infrastructure is necessary condition to use public transport); and that if \( r_p < r_n \), infrastructure provision could be reduced by \( C \pi r_n^2 - C \pi r_p^2 \), and by this reducing \( m_{\text{infra}} \) increasing utility. In Appendix A, we furthermore prove that the equality \( m_{\text{infra}} = m_p \) is true for exactly one value of \( r_p \).

We are now in position to determine the equations of optimal public transit provision. The social planner maximizes utility \( u(m_p) \) over \( m_p \) and \( r_p \) as given in Eq. 3 and integrated over \( r \) for given \( m_c \).
\[ L(m_c) = \max_{m_p, r_p} u(m_p, r_p) + \lambda(m_{\text{infra}} - m_p) \]

Hence, the first order conditions (FOC) are:
\[ \frac{du}{dm_p} = \lambda \left( 1 - \frac{dm_{\text{infra}}}{dm_p} \right), \]  
(8)
\[ \frac{du}{dr_p} = -\lambda \frac{dm_{\text{infra}}}{dr_p}. \]  
(9)

Together with the side constraint, these equations characterize optimal public transport provision. The first order conditions can be interpreted as follows. The first FOC (Eq. 8) essentially says that public transit provision is optimal, \( \frac{du}{dm_p} = 0 \), if the marginal change in public transit user charge translates into an identical change in marginal infrastructure costs, \( \frac{dm_{\text{infra}}}{dm_p} = 1 \).

To interpret the second FOC (Eq. 9), consider
\[ \frac{dm_{\text{infra}}}{dr_p} = \frac{d \left( \frac{C \pi r_p^2}{m_p} \right)}{dr_p} = \frac{C \pi (2D(r_p) - rD'(r_p))}{D^2(r_p)} \equiv \kappa(r_p) (2D(r_p) - rD'(r_p)). \]

The first term, \( \kappa(r_p) 2D(r_p) \), corresponds to the marginal increase in \( m_{\text{infra}} \) due to increased area coverage. The second term, \( -\kappa(r_p) rD'(r_p) \), corresponds to the marginal decrease in \( m_{\text{infra}} \) due to increased total distance traveled - inhabitants at the edge of the public transport area travel longer distances. The relative magnitude of these two effects decides on the sign of \( \frac{dm_{\text{infra}}}{dr_p} \). If the costs of additional area provision exceed the cost reduction effect due to total distance traveled, \( \frac{dm_{\text{infra}}}{dr_p} > 0 \) and the marginal utility with public transport area expansion is negative. Note that for the interpretation of the two
FOCs, we implicitly assumed \( \lambda > 0 \). This can be seen by looking at Eq. 9. If \( m_{\text{infra}} \) and hence total transport costs increase with distance, utility must decrease, whereas if \( m_{\text{infra}} \) decreases with distance, utility must increase.

The recursive and non-linear dependence of \( m_{\text{infra}} \) and \( m_p \) prohibit an analytical characterization of the equilibrium. We retreat to numerical investigation to characterize optimal public transit, analyzing first the social planner solution of the municipal government in 3.1 and 3.2, before investigating the market solution of public transit provision in 3.3.

### 3.2. Infrastructure provision shapes non-monotonous urban form

Optimal public transport provision can be numerically solved. The resulting city profile can be characterized by transport and rent costs, and density as a function of radial distance to the city center (Fig. 4). The transport cost increase with radial distance (top panel). At the private transport radius \( r_p \), the transport cost curve is non-monotonous and displays an step increase. Further outward unit rent costs decrease (medium panel). However, as lot sizes become larger, total rent per household increases. Households further outwards derive most of their utility from living amenities, i.e. lot size \( S \), whereas households further inwards have higher proportion of income that can be used for consumption. As an inverse of the plot size at distance \( r \), the population density \( \rho(r) \) decreases monotonously with radial distance, and also displays a non-linearity at \( r_p \). This jump is particular for our model. The infrastructure dependency of public transit means that public transit cannot substitute...
for car transport outside of $r_p$. Hence, the usual equilibrium condition $m_p r_p = F_c + m_c r_p$ does not hold anymore. Instead $r_p$ is characterized by the more encompassing trade-off between transport costs and land consumption: Those just outside of $r_p$ have considerably higher transport costs than those just inside of $r_p$, but can afford larger lot sizes paying much less per unit area.

As we have now established the model characteristics and understand the dependence of urban and public transport infrastructure on $m_c$ in particular instances, we are now in a position to analyze the impact of varying $m_c$.

3.3. Utility loss in the case of a monopolist

Consider a firm providing public transport infrastructure under profit maximization. The firm will choose $r_n$ and $m_p$ such that the profit

$$ P(r_n, m_p) = D(r_n)(m_p - m_{\text{infra}}(r_n)) $$

is maximized. Eq. 10 introduces the monopolist market solution. In this market solution, $m_p$ and $m_{\text{infra}}$ are not equal for non trivial $m_c$, i.e. $m_c > 0$. Note that for any $m_p$ it must be true that $m_p = F_c/r_n + m_c$. If $m_p$ were higher than car use would be more profitable at $r_n$ and the monopolist could decrease the public transit radius and increase her profit. The market solution is, generally, not optimal. Consider first the case when the profit $P$ is not part of the income of the urban residents, $Y = Y_0 + R$. Denote with $r_{pm}$ the public transit radius in the market solution, and $r_{ps}$ the public transit radius in the urban planner solution. With everything else being equal, in the market solution those inside $r_{pm}$ have to pay the complete mark-up costs for public transit $P$ but receive only part of the profit as income. Because of higher transit prices, land costs are slightly lower: more people choose to live outside of $r_{pm}$. But the lower land prices are insufficient to compensate for higher transit prices, as a similar location decision could also have been done in the urban planner solution. As it has not been taken, utility for people

![Figure 5](image-url)
living inside $r_{pt}$ is higher in the urban planner case than it is in the market case. Furthermore, as every inhabitant has identical utility, total utility is suboptimal in the market solution.

Now consider the case where the profit $P$ is part of the income of the urban residents, $Y = Y_0 + R + P$, see Fig. 1. This case is more complicated as higher transport costs are partially compensated by higher income, and hence utility improvements in consumption. In principle, one can substitute Eqs. 10 and 6 into Eq. 3 and compare utility in the urban planner and market case. However, to our best understanding an analytical solution is infeasible. Hence, we characterize how the divergence from the social optimum depends on fuel costs numerically. The result is depicted in Fig. 5. Clearly, for the parameter combination chosen in this paper, utility in the urban planner case is higher than for the market solution (congestion, air pollution and imperfect land markets further confound the picture). Ultimately, public transit infrastructure increases modal choice and location options for residents, by this increasing utility in reasonable circumstances.

The urban planer solution should not be confused with the social optimum. The step increase in transport costs at $r_{pt}$ (Fig. 4) suggests that everyone would profit from a further expansion of the public transit infrastructure. Indeed, Brueckner (2005) showed that transport subsidies can be warranted and achieve the social optimum if there are increasing returns to scale as is the case with public transport as formalized in Eq. 5. Such transport subsidies could be financed by land value taxes (Brueckner, 2005) or land value capture (Cervero, 1998).
4. Fuel prices determine urban form, modal shares and CO2 emissions

We have seen above that public transport provision leads to a distinct non-monotonous urban form. But the underlying factor, determining overall urban form and the optimal area for public transport infrastructure provision, is fuel prices, the marginal costs of car transport \( m_c \). Hence, indirectly, \( m_c \) determines which mode dominates the transport system.

Fig. 6 visualizes the impact fuel prices have on urban form, modal shares, and resulting CO2 emissions.

The impact of fuel prices on modal shares (and by this also on total transport costs and rent costs) and CO2 emissions is crucially mediated by the change of city size with costs of car transport. Specifically, for a fixed transport budget, city size is proportional to \( 1/m_c \): The city expands rapidly with falling fuel prices. For the more realistic case of a flexible budget, the city expands even faster (Fig. 6A). The reason is that residents compensate for an absolute increase in transport costs and concurrent decrease in spurious consumption by increased utility from land consumption. In other words, with decreasing car transport costs, the absolute costs of car transport, due to higher distances, increase faster than total costs of land consumption decrease.

Variations in fuel prices, \( m_c \) realize three different regimes of modal shares (Fig. 6B). For low \( m_c \), car transport is the exclusive mode, for intermediate prices, car and public transport coexist, for high \( m_c \), public transport is the exclusive mode.

An increase in fuel prices leads to a reduction in CO2 emissions, here assumed to be proportional to overall car distance travelled (Fig. 6C+D). First, an increase in fuel prices leads to a more compact urban form, reducing the average commuting distance. Second, an increase in population density increases the viability of public transit. Assuming, for illustration purposes, negligible CO2 emissions from public transport, this leads to further reduction in overall CO2 emissions. However, a quantitative comparison demonstrates that the first effect dominates the overall reduction in CO2 (Fig. 6D). In this model, modal shift only contributes less than 2% to overall reduction in CO2 emissions compared to the case when fuel prices are close to 0. With more realistic fuel prices modal shift plays a relatively more important role.

The next chapter visualizes that the explored relationships between fuel prices, urban form and modal shares are relevant in explaining tentatively aggregate data on global cities.

5. Urban transport in global cities

Newman and Kenworthy (1989, 1996) demonstrated a relationship between urban density and transport energy use on a set of global cities (Fig. 7A). The correlation is mostly driven by intercontinental differences: US, Australian, European, and Asian cities all cluster together, and the correlation within each cluster is much less significant than the correlation across clusters. Urban economics predicts a clear cut relationship between generalized transport costs, urban density, and resulting transport energy use. A calibration of the model to physical parameters allows to reproduce the inverse relationship between urban density (which is a function of \( m_c \)) and transport energy use (blue line Fig. 7A). While the urban economics framework is highly simplifying in assuming a monocentric city with homogeneous agents, the model still provides an intuition on the observed relationship.

In this light, the low urban density and high transport energy use of US-American cities is based in low \( m_c \), high income and an extensive road transport system which enables rapid unhindered transport in and between cities. A low \( m_c \) in the USA corresponds to very low fuel taxes and a well-developed road infrastructure, reducing monetary and time costs of car travel. European cities are historically denser, and provide less space for cars, thus increasing generalized transport costs, in addition to higher fuel taxes. Finally, inhabitants of Asian cities have, in average, lower income, by this raising the relative costs of fuel, and are living in very dense settlements which prohibit high car use.

Another observation of Reference Newman and Kenworthy (1996) is the inverse relationship between transport km of motorized vehicles and of public transit across cities (Fig. 7B). In this case, the model is less able to reproduce observed city data, systematically underestimating public transit km traveled (blue line, Fig. 7B). The main reason for this divergence is that the model assumes dense
Fig. 7. Urban form and transport demand of global cities. Data are from Kenworthy and Laube (2001). (A) Transport energy costs fall inversely proportional to urban density. Blue line: model prediction. Red line: model with linearized public transit network. The following parameters were used: \(a = 0.7, b = 1 - a, Y_0 = 10, \alpha = 1, F_x = 3, R_y = 0.02, \gamma = 5\). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

radial coverage of public transit. However, most public transit networks are linear in character, and develop along public transit axes. Obviously, such sparse networks cover longer distances. The arguments of the model still holds for such networks. Density still decreases with distance, but now with higher density public transit axes surrounded with lower density vehicle-served areas. The relative increase in total distance traveled in public transit is determined by the average transit corridor length and corridor thickness. This terms can be collapsed in a linearization factor \(\gamma\), determining the relative scale of public transit km covered. The general shape of this relationship can then be approximated by the model (red line in Fig. 7B, with \(\gamma = 5\)). The considerable variance displayed in Fig. 7B requires further investigation, possibly considering income effects, city size, a potential primacy effect (public transit of primary cities might be well subsidized), and other urban form characteristics.

6. Conclusions and discussion

We introduced a model of urban form and modal share, building on the Alonso-Muth-Mills framework. In this model, the economic feasibility and the spatial scope of public transit depends on urban form and \(m_c\). As a result, urban form is characterized by two modal areas, an inner city which is served by public transit, and an outer city which is served by car transport. Urban form displays a jump at the border between inner and outer city.

We derive the first-order condition, which reveals that the social optimal infrastructure provision is achieved when the marginal change in user charge is identical to the marginal change in infrastructure provision cost. We also demonstrate that a monopolistic private agent provides a suboptimal amount of public transport provision. The corresponding decrease in utility becomes particularly relevant for high marginal costs of car driving.

Crucially, fuel prices, \(m_c\), determine the city size, and by this urban form, and indirectly modal share. CO₂ emissions are reduced with increased fuel prices by (1) reducing the commuting distance and (2) enabling modal shift. In the static setting of this model, the first effect dominates the quanti-
tative outcome. Dynamic models, allowing for path dependencies in urban form, are likely to show that public transport infrastructures also influence urban form. The strong non-linear reduction of CO₂ emissions with increasing fuel prices suggests that fuel taxation would be particularly effective at low levels of fuel prices.

This conceptual framework provides an intuitive explanation of difference in viability of public transport across major world regions and can explain observation on the relationship between urban form and transport energy demand (Newman and Kenworthy, 1989; Newman and Kenworthy, 1996). For example, in the US low mc, available land and urban forms that are mostly unrestricted by historical pre-automobile developments allowed low-density development which makes public transport financially unviable and environmentally ineffective (Chester and Horvath, 2009) with exception of dense coastal city regions. In contrast, in Europa higher mc, limited land availability and historically denser cities restricted urban sprawl to some degree. As a result, public transport serves larger areas with acceptable subsidy levels. In Hong Kong, very high density and land value capitalization enables very high modal share of public transit and full recovery of infrastructure provision.

The AMM framework predicts that urban density is, ultimately, itself a function of transport costs. While this seems to be self-evident for urban economists (Pickrell, 1999), this insight is usually not part of empirical analysis. To further test this hypothesis, intertemporal data on income, fuel prices, road infrastructure and urban form need to be statistically analyzed in a dynamical model.

Clearly, the model framework is simplistic and not suitable for application to real cities. For example, in the model the population size is kept constant, and citizens are homogeneously receiving identical income. Urban travel is characterized one-dimensionally, i.e specifying a mono-centric rather than, e.g., a poly-centric city. Distinct transport corridors have a distinct impact on urban form (Anas and Moses, 1979). Nonetheless, while a monocentric model is highly unrealistic, the relationship between density and transport distance still holds in more complex real cities (Ewing and Cervero, 2010). Also, the setting is static, and neither path-dependence of transport infrastructures nor the question of time scales in transport and real estate markets is analyzed (e.g. Gusdorf and Hallegatte, 2007 address the importance of inertia in real estate markets in reaction to fuel price shocks). The modal choice is limited to two modes, ignoring numerous slow modes (walking, cycling, e-bikes), neither differentiating between different public modes (bus, tram, subway, regional train) nor between cars (electric cars, trucks). The utility estimation excludes the social value of climate change mitigation, air quality improvement and other common good provisions or co-benefits (Creutzig and He, 2009; Creutzig et al., 2012).

We suggest that the presented research promises further avenues for investigating sustainable urban form and addressing environmental pollution, energy security and climate change challenges. An extended framework can be useful to conceptualize co-benefits and public good provision of urban densification and transport pricing policies and elicit possibilities for financing these public goods.

Appendix A

Lemma Appendix A. 1. For any combination of (mc, Fc, C, u, Y, x), there is a unique rₚ in (0 rₙ] such that mₚ(rₚ) = minfra(rₚ).

We first demonstrate that for at least one value of rₚ, mₚ(rₚ) = minfra(rₚ). Second, we show that this equality is true for exactly one value of rₚ. To see the first part, observe that mₚ(rₚ) → ∞ for rₚ → 0, and mₚ(rₚ) → mₙ for rₚ → ∞. Also, as rₚ → 0, D(rₚ) decreases with degree < 3. Hence, mₚ(rₚ) → ∞ with degree > 3. Finally, as rₚ → 0, mₚ(rₚ) → ∞ (D(rₚ) remains finite as the population and the city radius rₚ is always finite). Hence, mₚ(rₚ) > minfra(rₚ) for rₚ → 0, and mₚ(rₚ) < minfra(rₚ) for rₚ → ∞. As both functions are continuous, they must cross at least once.

We turn to the second part of the proof. The slope of mₚ(rₚ) changes with −1. As mₚ(rₚ) has degree > 3, the slope of mₚ(rₚ) has always degree > 2. Hence, the difference mₚ(rₚ) − minfra(rₚ) is monotonically increasing in rₚ. We conclude that mₚ(rₚ) = minfra(rₚ) for exactly one rₚ.
References