Avoiding carbon lock-in: Policy options for advancing structural change

Linus Mattauch a,b,* , Felix Creutzig a,b , Ottmar Edenhofer a,b,c

a Mercator Research Institute on Global Commons and Climate Change, Torgauer Strasse 12-15, D-10829 Berlin, Germany
b Technical University of Berlin, Secretariat EB 4-1, Strasse des 17, Juni 145, D-10623 Berlin, Germany
c Potsdam Institute for Climate Impact Research, Postbox 601203, D-14412 Potsdam, Germany

ARTICLE INFO

Article history:
Accepted 6 June 2015
Available online xxxx

Keywords:
Structural change
Low-carbon economy
Carbon lock-in
Mitigation policies
Learning-by-doing

ABSTRACT

An obstacle for the transformation to a low-carbon economy is the carbon lock-in: fossil fuel-based (“dirty”) technologies dominate the market although their carbon-free (“clean”) alternatives are dynamically more efficient. We study the interaction of learning-by-doing spillovers with the substitution elasticity between a clean and a dirty sector to evaluate the robustness of policies averting the carbon lock-in. We find that the substitution possibilities between the two sectors have an ambivalent effect: although a high substitution elasticity requires less aggressive mitigation policies than a low one, it creates a greater welfare loss through the lock-in in the absence of regulation. The socially optimal policy response consists of a permanent carbon tax as well as a learning subsidy for clean technologies. We thus indicate that the policy implications of (Acemoglu, D., Aghion, P., Bursztyn, L., Hemous, D., 2012. The Environment and Directed Technical Change. American Economic Review 120 (1): 131–166), calling for merely temporary interventions based on the mechanism of directed technical change in the same setting, are limited in scope. Our results also highlight that infrastructure provision is crucial to facilitate the low-carbon transformation.

© 2015 Elsevier B.V. All rights reserved.

1. Introduction

Climate change mitigation requires drastic cuts in emissions in the 21st century and necessitates a transformation from a fossil-fuel based to a decarbonised economy. Both empirical evidence and theoretical argument suggest that an obstacle to this transformation is the possibility of a carbon lock-in (Unruh, 2000; Schmidt & Marschinski, 2009; Davis et al., 2010; Lehmann et al., 2012): the economy remains in an equilibrium in which carbon-intensive (“dirty”) technologies dominate the market, although they are intertemporally inferior to low-carbon (“clean”) alternatives. The size of such a market failure and the appropriate policy responses to it crucially depend on the substitution possibilities between such sectors, which are influenced by infrastructures, yet sometimes also by behavioural and institutional factors. They also depend on the mechanism underlying the development of clean production technologies. Which policy options best advance structural change towards the low-carbon economy is less clear; few studies have examined policy responses that are sufficient to avoid a carbon lock-in (Fisher & Newell, 2008; Gerlagh et al., 2009).

The purpose of this study is twofold: first, we contribute to quantifying the size of a lock-in by studying the impact of the substitution elasticity between a dirty and a clean sector. We find an ambivalent effect: a high elasticity creates a greater lock-in in the absence of regulation, but also requires less drastic policy intervention. This has implications for the effectiveness of second-best policy. Second, our article is a sensitivity study of Acemoglu et al. (2012) (henceforth: AABH), who analyse the impact of directed technical change in the framework of the present article: our results show that with learning-by-doing behaviour of clean technologies instead of directed technical change, effective mitigation policies need to be permanent, not temporary, regardless of the value of the substitution elasticity because demand for intermediate dirty production never becomes zero.

We use a two-sector intertemporal general equilibrium model and solve it numerically to identify policy options that are sufficient to avoid high welfare losses. A common stylized setting is employed to depict structural change to a low-carbon economy: there is one clean sector, without emissions, and one dirty, emitting greenhouse gases. This approach has been adopted by AABH and, for instance, also by Gerlagh & Hofkes (2002) and Cassou & Hamilton (2004). Our model setup, including the representation of global warming, is nearly identical to that of AABH in order to be comparable in terms of policy implications: we respect all parameter choices and functional forms of AABH except those concerning the nature of technological progress. While AABH focuses on the effects of directed technical change for the transformation to a low-carbon economy, our work relies on the assumption of learning through spillover effects in the clean sector as its capacity is built up.

Such a learning-by-doing approach (Arrow, 1962) is well-established within energy economics (Kverndokk & Rosendahl, 2007): the cost of renewable technologies decreases with cumulative installed
capacity at a stable rate (Fischedick et al., 2011, Ch.10.5.2). No comparable effect exists for dirty, mature technologies (McDonald & Schrattenholzer, 2001). It has moreover been demonstrated theoretically that – in presence of learning-by-doing externalities – optimal carbon pricing is insufficient to overcome a lock-in into mature low-carbon technologies in the energy market (Kalkuhl et al., 2012). We further discuss the differences between the assumptions of learning through increased capacity and directed technological change and their empirical plausibility in Section 2.1.3.

The carbon lock-in was originally examined from a systemic perspective highlighting the co-evolution of technology and institutions (Unruh, 2000): the technologically caused lock-in is exacerbated by institutional and policy failures. Our analysis focuses exclusively on the lock-in as a phenomenon of market failure and leaves aside institutional failures. In our model the lock-in arises through the combination of two externalities: first, learning spillovers that arise from building up capacities in the clean sector are unappropriated and are a stylized representation of positive externalities in the development of low-carbon technologies. Second, the negative effect of carbon-intensive production on utility through climate damages are ignored in the unregulated market outcome. The combination of the externalities can prevent the market from building-up the carbon-free sector and cause a delayed transition to the low-carbon economy. Different interpretations of the concept of a carbon lock-in are frequent (Lehmann et al., 2012; Page, 2006), with some focussing on the non-malleability of capital, for instance by irreversible investment in coal power plants, as an additional cause of the suboptimal share of clean production. Instead we focus here on the interplay between one cause inhibiting the development of the clean sector and the substitution possibilities: as the latter represent infrastructural and institutional limitations to produce clean instead of dirty goods, it is the interplay of both factors that captures the co-evolution of technology and institutions. Since the specific model setup matters for analysing the carbon lock-in, we rely on numerical solutions instead of using an even more stylised model that would be more amenable to analytical treatment. The model is described in Section 2.

The principal message of our study is that although a higher substitution elasticity requires less aggressive optimal mitigation policies, it creates higher welfare losses from a lock-in. The optimal policy response requires both a carbon tax and a learning subsidy. The ambivalent role of the substitution possibility suggests to also examine second-best policy responses: we show that even if the only policy option available is a carbon tax, it can correct most of the welfare loss from the lock-in if the tax is set much higher. Furthermore, regarding the sensitivity of the results of AABH with respect to their conception of technological progress, we find that whether climate change mitigation requires a permanent or a merely temporary policy intervention depends primarily on the mechanism governing technological progress in the clean sector and not on the value of the substitution elasticity. We show that the optimal policy suggested by AABH, which is temporary and triggers a rapid switch from the carbon-intensive to the low-carbon sector, does not reproduce the socially optimal outcome in our model, which differs only by the assumptions about the technologies. Instead, effective mitigation policies need to be permanent, regardless of the value of the substitution elasticity. This is because with a somewhat more gradual development of clean technologies, there will be permanent demand for dirty production that decreases but is never strictly zero. Further, substitution possibilities crucially influence the feasibility of different climate policy options: we find that more stringent mitigation targets require a (much) higher carbon tax if the elasticity is low. They also determine the timing of the optimal subsidy to the clean sector.

The topic of this article is thus related to, but independent of, discussions about adverse effects of green subsidies on climate change mitigation along the lines of a “Green Paradox” (Sinn, 2008, 2012). The idea of the Green Paradox (in the present context) is that green subsidies may provide an incentive for resource owners to extract a part of their fossil reserves earlier because the subsidies may devalue their assets. Whether this effect matters for climate change mitigation has been debated (van der Ploeg, 2013; Edenhofer & Kalkuhl, 2011): climate change mitigation in this century depends crucially on achieving a limit on cumulative emissions much lower than the total emissions that would be generated from burning all fossil resources. Green subsidies, in particular, may lead to a temporarily higher extraction of fossil resources, but will also decrease future resource extraction. They thus lead to more resources being left underground and the latter effect is likely to dominate the former (van der Ploeg, 2013). To focus exclusively on the specific lock-in effects due to the substitution elasticity and for comparison to the study by AABH, we abstract from these effects by not explicitly considering resource owners in our model, which would be necessary to generate effects similar to the Green Paradox. The reason is that effects related to the substitution elasticity and learning behaviour of clean energy are independent of the timing effects of resource extraction due to anticipation of policy changes by resource owners that give rise to the Green Paradox. In contrast, recent research explicitly takes into account fossil fuel extraction to also consider the optimal policy mix of carbon taxation and subsidising renewables (Rezai & van der Ploeg, 2013) or the second-best case of subsidising renewables when carbon pricing is infeasible (van der Ploeg & Withagen, 2014). However, these articles do not adopt a two-sector structure.

A substitution elasticity is not a natural constant, but an artefact of economic theory: the ease of using one technology or product instead of another one. In particular, substitution possibilities are influenced by infrastructure in relevant sectors of the economy, although behavioural and institutional effects are also important, for instance, in the transport sector, too. In the electricity sector, in which the division between carbon-free and fossil-fuel based technologies is clear-cut, the use as opposed to the generation of renewable energy is not straightforward and requires appropriate infrastructure since renewable energy production misaligns with electricity demand in time and space. Infrastructure investments can enable renewable energy use so that the misalignment across space and time is compensated for: grid extensions allow large scale transfers of electricity from generation sites to load sites. In the transport sector, substitution possibilities can also be mostly understood in terms of technology and infrastructure (Schafer et al., 2009). However, consumer preferences are also important to determine the elasticity between carbon-intensive and low-carbon modes in the case of transportation, since mode choice also involves important trade-offs in terms of security, privacy, comfort and health as well as being driven by habituation to a single mode. Both examples highlight the need for additional policy that increases the elasticity, for example financing appropriate energy infrastructure or fostering institutional changes towards intermodal transport. We suggest that the investments in these infrastructures can be interpreted as an increase in the substitution possibilities. Thus a scenario of an increasing substitution elasticity is the most plausible scenario for the coming decades, particularly in the light of estimates that current substitution possibilities between clean and dirty sectors are very low (Pelli, 2011; Pottier et al., 2014).

2. Model

We use a discrete-time intertemporal general equilibrium model that is similar to that of AABH except for the different conception of technological progress and the different role of government policy options. There are two sectors, one emission-intensive (“dirty”) and one carbon-free (“clean”). Those sectors manufacture inputs used in the production of a final good that can be freely used for investment in each sector or for consumption. Households ignore the effect of global warming, which is described by the heuristic approximation chosen in AABH. Technological progress in the clean sector is subject to a
learning-by-doing effect based on its cumulative capacity, technological progress in the dirty sector is exogenously given.

The decentralized equilibrium contains two market failures: first, the environmental externality – dirty production decreases utility through damages of global warming – is not taken into account by the decentralized agents. Second, firms in the clean sector do not appropriate the intertemporal learning spillover resulting from their production.

2.1. The decentralized economy

We present the maximization problems of the agents in the economy and how policy instruments enter their choices. The derivations of the first-order conditions are given in Appendix A.

2.1.1. Demand

The representative household derives utility \( U \) from consumption \( C \) and the environmental quality, represented as the size of the carbon sink \( S \):

\[
U(C, S) = \frac{\phi(S) C^n}{1 - n} - \frac{1}{1 - \eta}
\]

with \( \eta \neq 1 \). The function \( \phi(S) \) represents the impact of climate damages on utility including the possibility of an environmental catastrophe and is specified in Section 2.1.4. The household maximizes intertemporal utility, which is given by:

\[
\max_{C, K_t} \sum_{t=0}^{\infty} \frac{U(C_t, S_t)}{(1 + \rho)^t}
\]

It is assumed that the effect of the investment decisions on \( S \) is ignored, thus representing climate change as an externality. The household owns labour \( L_t \) and capital \( K_t \) and faces the budget constraint

\[
C_t + I_t = r_t K_{t+1} + r_t K_{t+2} + w_t L_{t+1} + w_t L_{t+2} + \Gamma_t,
\]

with \( r_t \) denoting investment, \( r_t \) the interest rate, \( w_t \) the wage in sector \( t = 1, 2 \) and \( \Gamma_t \) the lump-sum transfer from the government budget to the household. The price of consumption is set to one as the constraint on the capital stock

\[
K_{t+1} = L_t + (1 - \delta) K_t
\]

with depreciation rate \( \delta \). The household can distribute labour and capital arbitrarily between the sectors:

\[
\bar{L} = 1 = L_{1,t} + L_{2,t}
\]

\[
K_t = K_{1,t} + K_{2,t}.
\]

Labour is normalized to 1 for simplicity.

2.1.2. Supply

The economy produces a single good \( Y \), which is composed of a carbon intensive intermediate good \( Y_1 \) and a low-carbon intermediate good \( Y_2 \). It is assumed that this final production is given by a CES function

\[
Y(Y_1, Y_2) = A Y_1^{\hat{\gamma}} Y_2^{1 - \hat{\gamma}}
\]

in which \( \hat{\gamma} > 0 \) represents the elasticity of substitution between the clean and dirty goods. The higher the elasticity \( \hat{\gamma} \), the better substitutable are the clean and the dirty good. It may change with time exogenously. This seems plausible over long time horizons as substitution possibilities are driven by appropriate infrastructure, so that they can be changed by suitable policies. For instance, these may include to give other urban transport modes priority over cars or to adapt national power grids to the requirements of a high share of renewable energy generation.

General technological progress \( A_t \) evolves exogenously, reflecting an exogenous growth rate or total factor productivity \( g_c \):

\[
A_t = A_0 \exp(g_c t).
\]

Intermediate good \( Y_1 \) is produced from capital \( K_t \) and labour \( L_t \) according to a Cobb–Douglas production function:

\[
Y_1 = F_1(K_1, L_1) = K_1^\alpha L_1^{1 - \alpha}
\]

\[
Y_2 = F_2(A_2, K_2, L_2) = K_2^\beta L_2^{1 - \beta}.
\]

2.1.2.1. Final-good producer. The final good producer maximizes profits \( \Pi \):

\[
\max_{Y_1, Y_2} \Pi = Y - (p_1 + \tau_t) Y_1 - p_2 Y_2
\]

where \( p_1, p_2 \) are the prices of the clean and dirty goods and \( \tau_t \) is a tax on emission-intensive products (carbon tax). The carbon tax is levied on the final good producer to reflect the fact that in principle any product could generate emissions, although levying it on the producer in the dirty sector would of course be equivalent.

2.1.2.2. Dirty sector. The dirty firm maximizes profits \( \Pi_1 \):

\[
\max_{K_{1,t}} \Pi_1 = p_1 Y_1 - r_1 K_1 - w_1 L_1.
\]

2.1.2.3. Clean sector. In the clean sector, there is additional endogenous technological progress that depends on the cumulative output of that sector through learning-by-doing (Arrow, 1962). The cumulative output represents the stock of experiences made and is thus formalized as:

\[
H_{t+1} = (Y_{2,t} - Y_{2,t-1}) + H_t,
\]

\( H_t \) representing the initial stock of knowledge. The technology of the sector is given by

\[
A_{2,t} = \frac{\beta}{1 + (\rho \beta)^t}
\]

so that \( A_{2,t} \to \beta \) as \( H_t \to \infty \). The choice of making learning-by-doing dependent on output and not capital changes has also been adopted by Kalkuhl et al. (2012): learning rates are standardly estimated given changes in cumulative installed capacity, which is related to physical output. It is matched closer by output in monetary units rather than capital in monetary units for two reasons: first, in the highly stylized setting of two sectors, capital will need to be broadly interpreted and include other things besides installed capacity, for instance the machines needed to produce and install it. Second, because more output can be produced for an additional unit of capital investment through the learning, capital invested later (after learning) is more productive than older capital, necessitating more complicated functional forms.

Further, the functional form in Eq. (12) is justified as follows: \( \beta, \gamma \) and \( \alpha \) determine the shape of the learning curve for the clean technology. The level the clean technology converges to when it reaches maturity is given by \( \beta \), thus determining the maximum productivity. The speed of the convergence to that level is determined by \( \omega \) and \( \gamma \). The three parameters together determine the learning rate of the technology. It is additionally assumed that \( H_0 \) is small, so
that technology in the dirty sector is initially much more advanced. Moreover, we assume \( b > A_{1,0} \) in accordance with expectations on efficiency of renewable technologies in the future (Breyer & Gerlach, 2013; Kost & Schlegl, 2012). Clean technology thus lags behind and takes more time to develop, but will eventually be more advanced than dirty technology. On this high level of generality the chosen learning curve of the clean technology in the model cannot correspond to actual data of the learning behaviour of renewable energies. Yet the functional form employed is commonly used for the learning behaviour of carbon-free technologies (Kalkuhl et al., 2012) and similar modelling of learning-by-doing is very common in energy economics (Kverndokk & Rosendahl, 2007; Edenhofer et al., 2005).

It is assumed for simplicity that spillovers in the clean sector are totally appropriated by firms: individual firms are small enough not to take into account their individual contribution to the stock of global knowledge. See Romer (1986) and Fisher & Newell (2008) for a more sophisticated treatment.

The clean firm maximizes profits \( \Pi_2 \):

\[
\Pi_2 = (p_2 + \tau_2)Y_2 - r_2K_2 - w_2L_2.
\]

Here \( \tau_2 \) is a subsidy on clean output. No capital inertia in investments is assumed: in equilibrium, wages and interest rates are equalized across sectors because production factors are perfectly mobile. Thus

\[
w_1 = w_2 \tag{13}
\]

and

\[
r_1 = r_2. \tag{14}
\]

These conditions hold as production is never zero in these sectors, which is the case because intermediate inputs are imperfectly substitutable. Next, the development of the clean and dirty technology chosen here is compared to the mechanism propounded by AABH.

### 2.1.3. Two conceptions of technological progress: learning-by-doing vs. directed technical change

Technological progress that differs between the two sectors of an economy leads to different sectoral growth rates (Baumol, 1967). For the case of structural change towards the low-carbon economy AABH focus on endogenously determined sector-biased technological progress — directed technical change (DTC) (Acs & Audretsch, 1990). Our approach is to assume a sector-biased technological change that is exogenously given, but inspired by empirical findings on the learning of low-carbon technologies. Here we compare the two approaches and discuss their relevance.

In the model of AABH, profit-incentives of workers in research and development (“scientists”) determine whether technological progress proceeds in the clean or dirty sector. However, the model has the feature that innovation occurs in one sector only, unless a knife-edge condition (Lemma 1 of AABH) is fulfilled: as a consequence, if the clean sector is significantly more productive than the dirty sector, no dirty output will be produced and the whole workforce will work in the clean sector and vice versa. This is due to the specific calibration of the direction of technical change: from Eq. (11) and the specified calibration (\( \gamma = 1; \eta_1 = \eta_c = 0.02 \)) in Section V of that study, one can deduce that if all innovation improves the technology of the dirty sector \( A_1 \) and none that of the clean sector \( A_2 \):

\[
A_{1,t+1} = 1.02A_{1,t-1} \quad \text{and} \quad A_{2,t+1} = A_{2,t-1}. \tag{15}
\]

If instead all innovation benefits the clean sector \( A_2 \):

\[
A_{1,t+1} = A_{1,t-1} \quad \text{and} \quad A_{2,t+1} = 1.02A_{2,t-1}. \tag{16}
\]

The former is the case if the dirty sector is much more advanced than the clean sector. The latter happens if there are sufficiently high subsidies for the clean sector. The switch between these two possibilities – the time that passes while all scientists “migrate” from one to the other sector – is either immediate or happens within a time span of 15 years (3 periods) in AABHs numerical simulation, depending on different parameter combinations.

By contrast, we examine the impact of a well-documented stylized empirical fact about low-carbon technologies on policies for advancing structural change: a decline in cost per unit output through learning effects due to increased experience (Fischelidick et al., 2011; Kost & Schlegl, 2012). No learning effect, at least not on a similar scale, is known for mature, dirty technologies (McDonald & Schrattenholzer, 2001; Breyer & Gerlach, 2013), in particular as in this model, \( A_1 \) also reflects the cost of fossil fuel. While in our model there is an overall increase in total factor productivity that affects both sectors equally, technology in the clean sector \( A_2 \) depends positively on cumulative capacity \( H_t \) that represents the stock of experiences made with the low-carbon technology (see Eq. (12) in Section 2.1.2 for a detailed explanation.) Examining the impact of learning-by-doing mechanisms in the structural change framework adopted by AABH has already been called for Pottier et al. (2014).

Which conception of technological progress is more plausible for modelling structural change at this abstract level? While there is some evidence for the concept of DTC (Popp, 2002), the decisive factor for understanding the risk of intertemporal lock-ins during the transition to the low-carbon economy is the learning behaviour of renewable energies (Edenhofer et al., 2011). Learning-by-doing is also the more comprehensive concept: it includes the migration of scientists and engineers to other sectors. Scientists and engineers need experiments to learn and need to build up capacities and equipment — this cannot be steered by huge research subsidy over a short time period as a suddenly much larger output of the clean technologies might not be profitable in a very short time span. The learning-by-doing approach thus stresses that the redirection of R&D-efforts is subject to path-dependencies in the careers of individuals, in the technological regulations and in the design and management of research institutions.

Our aim in this study is not to doubt the importance of DTC for understanding economic growth and structural change. We argue instead that the particular mechanism of DTC in the model of AABH represents a special case in the space of possible structural transformations towards the low-carbon economy, conflicting with empirical studies of the learning behaviour of technologies. We identify an immediate switch between sectors as produced by AABHs model as corresponding to a very high learning curve in our model (see Section 3.4). Our study hence shows that focusing on the learning behaviour of low-carbon technologies changes the picture of sensible policy responses to the carbon lock-in.
The size of the carbon sink evolves as follows:

$$S_{t+1} = -\xi Y_{1,t} + (1 + \xi)S_t. \quad (17)$$

if the right-hand side is between 0 and the pre-industrial level $S$. Alternatively, $S_{t+1} = 0$ if the right-hand side is negative, and $S_{t+1} = S$ if the right-hand side is greater than $S$. $S$ is subsequently related to global mean temperature. Assume the standard approximation that if $\Delta$ is global mean temperature and $C_{CO2}$ is carbon concentration in the atmosphere, then

$$\Delta = 3 \log_2 \frac{C_{CO2}}{280} \quad (18)$$

which, for instance, implies that a doubling of CO$_2$ concentration leads to a temperature increase of 3°C. From this equation, one can relate $S_t$ and $\Delta$ in two steps:

First, calibrate the range of $S$:

$$S = 280 - 2^{\frac{\Delta}{T}} - \max(C_{CO2}, 280). \quad (19)$$

$S$ is zero (using Eq. (18)) at the temperature level of an “environmental disaster”, which is here given by $\Delta_{dis}$. So if $\Delta = \Delta_{dis}$, utility is zero (see Eq. (22)). Further, $S$ reaches its maximum value for the pre-industrial value of $C_{CO2}$.

Second, $\Delta$ can then be expressed as a smooth function of $S$ if $C_{CO2} \geq 280$: from Eq. (19), it follows for this case that

$$C_{CO2} = 280 - 2^{\frac{\Delta}{T}} - S \quad (20)$$

and thus using Eq. (18) that

$$\Delta(S) = 3 \log_2 \left(2^{\frac{\Delta}{T}} - \frac{S}{280}\right) \quad (21)$$

Finally, define a damage function $\phi(S)$ (introduced above as the argument of the utility function) that gives utility the desired property of a possible environmental disaster and follows appropriate standard assumptions about climate damages for a moderate temperature increase. The function employed by AABH that has these properties is

$$\phi(S) = \varphi(\Delta(S)) = \frac{(\Delta_{dis} - \Delta(S))^{2\lambda} - \lambda \Delta_{dis}^{2\lambda}(\Delta_{dis} - \Delta(S))}{(1 - \lambda)\Delta_{dis}}. \quad (22)$$

This function ensures that the marginal utility of the carbon sink tends to $-\infty$ and utility tends to $-\infty$, as $S$ goes to zero. The parameters are chosen to match the standard damage function of the DICE model (Nordhaus, 1994) for the medium range of possible temperature increases (for details see AABH, Sections I. and V A. and its FEEM working paper version [Nota di Lavoro 93.2010], Section 4). In particular, $\phi = 1$ if $S$ is at its pre-industrial level $\bar{S}$, so that $\phi$ represents damages in percent of remaining consumption.

2.2. Social optimum and deriving optimal policy: analytic results

We distinguish two types of equilibria: the social optimum and the decentralized equilibrium with government intervention introduced in the previous subsection (which includes the laissez-faire case if there is no policy in place). We obtain our results by comparing different policy choices of the government with the social optimum. Here we derive expressions for the socially optimal policy instruments in terms of the shadow prices of the socially optimal solution. In the next subsection, we discuss a different approach for numerical implementation, which allows to calculate second-best scenarios and which is used for the numerical results in Section 3 (see van der Ploeg & Withagen, 2014; Kalkuhl et al., 2012 for similar recent treatments of the two approaches).

Determining the social optimum provides a benchmark for evaluating the effectiveness of policy options. The social planner determines the optimal allocation in the economy by maximizing intertemporal utility of the representative agent subject to the constraints on factors of production, the production technologies, the influence of environmental quality and the macroeconomic budget constraint. The social planner problem is thus

$$\max_{t=0, \ldots, T} \sum_{t=0}^{T} U(C_t, S_t) \quad (23)$$

subject to Eqs. (4)–(12), (17)–(22) and the macroeconomic budget constraint $V_t = C_t + I_t$.

The social planner hence recognizes both the negative impact of the carbon-intensive production on the household’s utility through decreasing environmental quality as well as the productivity gains through learning-by-doing in the clean sector. The full optimization problem and the optimality conditions are presented in Appendix B.

Here we derive expressions for the socially optimal levels of the tax and the subsidy in terms of the shadow prices associated with the social planner problem by calculating the social value of clean and dirty production in terms of the co-states and marginal productivities. The optimality conditions are given in Appendix B as Eqs. (B.10)–(B.17) (as they are not employed in deriving the numerical results below, see Subsection 2.3). They can be interpreted as follows: $\lambda_t, \mu_t$ and $\kappa_t$ are the shadow prices for additional units of total capital, environmental quality and knowledge, respectively. This means that $\lambda_t$ stands for the social cost of carbon (in utilities), $\mu_t$ for the social benefit of capital and $\kappa_t$ for the social benefit of knowledge, meaning that marginal cost or benefit of increasing the respective flow quantities by one unit, as usual, $\nu_t$ and $\psi_t$ are shadow prices corresponding to the constraints given by the production functions of the intermediate goods, representing the social benefits that would result from relaxing the respective production functions.

Eq. (B.10) shows that the social cost of consumption $\mu_t$ is equal to the discounted marginal utility of consumption, as usual. Eq. (B.11) provides a difference equation for the social cost of carbon $\lambda_t$ and shows that its change depends on the discounted marginal utility of the atmosphere and its regeneration rate. Eq. (B.12) similarly gives the change of the social cost of consumption $\mu_t$, the difference to its decentralized version (see Eq. (A.2)) is that it specifically depends on the value of marginal productivity of capital in the second sector (instead of the interest rate), due to the way the interdependencies that only the social planner takes into account are represented in the optimization.

Eq. (B.13) can be rewritten as follows

$$\frac{\nu_t}{\mu_t} = \frac{\partial Y}{\partial Y_{1,t}} - \xi \frac{\lambda_t}{\mu_t} \quad (24)$$

and thus provides an expression for the socially optimal pricing of the dirty intermediate good. When normalized with respect to the social cost of consumption, $\nu_t$ gives the social value of an additional unit of the dirty intermediate good (in monetary units), which is the difference of its marginal productivity in final production and the social value of the emission associated with it. One can thus determine the optimal carbon tax level in terms of the respective shadow price. By comparing Eqs. (24) and (A.4) one obtains:

$$p_{1,t} = \frac{\nu_t}{\mu_t} \quad \text{and} \quad \tau_{1,t} = \xi \frac{\lambda_t}{\mu_t} \quad (25)$$
Similarly, to obtain an expression for the optimal subsidy and the price of the clean good, rewriting Eq. (B.14) yields

$$\psi_t = \frac{\partial Y}{\partial Y_{2t}} + \frac{n_{t+1}}{\mu_t} \psi_{t+1}. \quad (26)$$

Inserting Eq. (B.17) one obtains an expression for the socially optimal pricing of the clean intermediate good:

$$\psi_t = \frac{\partial Y}{\partial Y_{2t}} + \frac{1}{\mu_t} \left( \psi_{t+1} \frac{F_2}{\partial g} \frac{dg}{dH_t} \right). \quad (27)$$

Thus, \(\psi_t\), when normalized, indicates the social value of an additional unit of clean intermediate output, which is the sum of its marginal productivity in final production and the value of the additional stock of knowledge generated by it which is productive in the next period. One can thus determine the optimal level of the subsidy and the price of the clean good in terms of the shadow prices, by comparing Eqs. (27), (A.5), (A.8) and (A.9):

$$p_{2t} = \frac{\psi_t}{\mu_t} \text{ and } \tau_{2t} = \frac{1}{\mu_t} \left( \psi_{t+1} \frac{F_2}{\partial g} \frac{dg}{dH_t} \right). \quad (28)$$

Finally, by comparing Eqs. (B.15) and (B.16) with Eqs. (A.6)–(A.9), one recovers the relationship between the prices that govern the distribution of capital and labour in terms of the appropriate shadow prices.

### 2.3. Model implementation and calibration

The optimisation problem of the social planner (Eq. (23)) and the government (Eq. (30) below) form non-linear programs. These are solved numerically with GAMS (GAMS Development Corporation, 2008), using its solver CONOPT: for the case of the social planner, GAMS solves the optimisation problem given by Eqs. (4)–(12), (17)–(22) and (23) directly, without the need for considering its first-order conditions.

The decentralized equilibrium is treated differently: the laissez-faire equilibrium is completely described by the first-order conditions of household and firms as well as the constraints on technologies, production and budgets: (3)–(22), (29) and (A.2)–(A.9). This set of difference equations can be solved by optimising with respect to a dummy variable. For the cases of government intervention, GAMS optimizes the household’s welfare (see Eq. (30)) with respect to all the equations of the decentralized equilibrium and with the policy instruments as decision variables.

In more detail, this means that the government anticipates the learning subsidy – the carbon tax and the learning subsidy – with the aim of maximising social welfare. It redistributes taxes and subsidies lump-sum to the representative agent:

$$\Gamma_t = \tau_{1t} Y_{1t} - \tau_{2t} Y_{2t}. \quad (29)$$

The maximisation problem of the government is thus

$$\max_{\tau_{1t}, \tau_{2t}} \sum_{t=0}^{T} U(C_t, S_t) \frac{1}{(1+\rho)^t} \quad (30)$$

subject to the first-order conditions of the agents, the household’s budget constraint, the technology constraints and the state of the carbon sink, that is, subject to Eqs. (3)–(14), (17)–(22), (29) and (A.2)–(A.9). This approach means that the first-order conditions of households and firms serve as a reaction function for the government’s optimization problem, which optimizes welfare. In contrast, the social planner solution only serves as a benchmark to assess the goodness of policy options.

Finally, second-best analyses are then conducted by limiting the government’s possibilities for intervention to one variable only. The approach just detailed is essential beyond the method employed in Subsection 2.2 for computing the second-best scenarios below.

The model is calibrated to be comparable with AABH. The time period of the numerical simulation corresponds to five years, intertemporal parameters are hence chosen with respect to that interval. The time horizon is \(T = 175\) years.\(^2\) Those parameters with values identical to the calibration of the model of AABH are displayed in Table C.2 in Appendix C. Since the technological progress and capital dynamics are conceptualised differently from AABH in the present model, a standard rate of capital depreciation (0.03% per year, that is \(\delta = 0.141\)) is selected and the rate of exogenous technological progress is chosen to obtain a long-run growth rate of consumption of 1.8% per year. The remaining parameters for the technological progress in the clean sector are chosen such that the clean sector initially lags behind and eventually is more efficient than the dirty sector, so that technological progress reproduces the stylized facts about future learning of renewable technologies (Fischedick et al., 2011, Ch. 10.5.2).

### 3. Numerical results

In this section the results of the numerical simulations of our model are reported. As the model has two externalities – pollution and technology spillovers – two policy instruments are needed to reach the social optimum (first-best). A policy with just one instrument cannot achieve the first-best, and so is a second-best policy. First the size of the lock-in is quantified (3.1), subsequently the optimal policy intervention is calculated as the welfare-maximizing tax paths (3.2). High welfare losses can be avoided even if only a single instrument is available to the government (3.3). The impact of a higher learning curve on the duration of the structural change is examined (3.4). Finally, the optimal policy is characterized when the social optimum is constrained by a two degree target (3.5).

Throughout, three cases for the substitution possibilities for clean and dirty production are considered and represented by values of the substitution elasticity \(\epsilon\). Two cases, \(\epsilon = 3\) and \(\epsilon = 10\), are equal to those in AABH’s numerical simulation to make our findings comparable to that study. In addition a third case is examined in which \(\epsilon\) increases linearly over time from initially \(\epsilon = 3\) up to \(\epsilon = 10\) eventually, the computationally simplest case of an increasing elasticity. This case represents future infrastructure developments designed to facilitate the use of clean instead of dirty goods. The three cases are labelled the “low”, “high” and “increasing” scenario below. Welfare losses resulting from sub-optimal or missing policy intervention are quantified in balanced-growth equivalents (BGE) (Mirrlees & Stern, 1972; Anthoff & Tol, 2009) as environmental quality enters utility directly in our model. A balanced-growth equivalent for policy \(P\) is the level of initial consumption that, if that consumption grows with a constant rate, produces the same welfare as does \(P\). We follow the computations in Anthoff & Tol (2009), Section 2.1. in our numerical implementation to calculate BGEs for the policy scenarios below.

The results remain limited in so far as the model remains a crude representation of the real world dynamics of a carbon lock-in. In particular, optimal warming levels are to be understood in this way — for making points about qualitative differences between scenarios, not about exact implications of potentials goals of climate policy. For the purpose of this article, making theoretical points about the impact of a combination of assumptions about technological progress and substitution elasticities on the lock-in, a close calibration to real-world data is inessential.

---

\(^2\) The model is solved with a time horizon of \(T = 250\) years, but since towards the end of that time horizon the deinvestment dynamics dominate economic behaviour, we do not show the last 75 years in the illustrations below, as is standard.
intertemporally inferior carbon-intensive technology dominates the market and a transition to the low-carbon economy occurs later than would be socially optimal. For a high substitution elasticity the lock-in is generally more severe than for a low one: with a high elasticity there is less demand for clean production so the learning in that sector takes longer and aggravates the market failure. The lock-in into the dirty sector is quantified in three respects: (a) the aggregate discounted welfare losses over time are measured in balanced-growth equivalents, (b) the delay of the structural transformation is given as the difference between the socially optimal and actual time of reaching a 50% share of the clean sector and (c) the total and the additional amount of global warming compared to the social optimum is calculated. For the different substitution scenarios the simulations can be summarized as follows: the “high” scenario leads to a much greater lock-in compared to the “low” scenario on all scales, whereas the “increasing” scenario represents an intermediate case, see Fig. 1. Fig. 2 compares the socially optimal time paths with those of the unregulated market outcome for the share of the clean and the dirty sector and the state of the atmosphere (the time paths of further variables are presented in Appendix D). There are significant differences in the deviations of the decentralized paths from the socially optimal outcome: in the “low” case a share of the clean sector is missing that is approximately constant over time, while in the “high” case the switch from the dirty to the clean sector is delayed, the “increasing” scenario representing a middle case. The socially optimal amount of global warming is below 2 °C for the “high” and “increasing” case and 2.9 °C for the “low” case: due to the difficulty in substituting away from low-carbon production, it is socially optimal to accept more global warming in order to have more consumption. However, the better the substitution possibilities, the higher is the additional amount of global warming produced by the externality. The sectoral shares in the “increasing” case are, moreover, non-monotonic because the changing substitutability influences the demand for the intermediate goods: as long as the clean technology is not yet mature, an increasing substitution elasticity leads to less demand for the clean good, while the opposite is true when the clean technology is more competitive then the clean good.

3.2. First-best policy response

In the absence of policy intervention, severe welfare losses occur due to the combination of market failures that creates the lock-in. This motivates the subsequent analysis of policy responses that avoid it. To correct the externalities, a carbon tax and a learning subsidy are feasible policy instruments. The welfare-maximizing time paths of the policy instruments are computed for the three different substitution possibilities (Fig. 3). In all cases, carbon prices are increasing with time, and, with the exception of the “high” case of \( \epsilon = 10 \), subsidies are decreasing. Carbon prices increase because of general productivity growth and higher damages. The share of the dirty sector (strongly) decreases in all scenarios, while its absolute volume (moderately) increases only in some. The subsidy for the case of \( \epsilon = 10 \) is non-monotonic for the following reason: initially, subsidising the clean sector is not necessary because the benefit of this would be too low (the sector being too unproductive to warrant a subsidy). It is sufficient if the subsidy starts later for the clean sector to dominate later on — which indicates that for such substitution possibilities the switch from one dominating technology to the other can be very fast. For this case the subsidy strongly increases once the government starts to use it and peaks around two decades after its introduction. By comparison, the optimal policy in the “low” case of \( \epsilon = 3 \) involves a substantially higher carbon price and a moderately higher learning subsidy (at least after the sudden initial increase and peak of the subsidy in the “low” case) than in the other cases. Except for an initially high learning subsidy, the “increasing” case requires policy instruments much more in the order of the “high” case than the “low” case.

The optimal policy intervention for the structural change to avoid the lock-in is permanent for all scenarios. This is in particular true for the carbon tax: carbon pricing needs to be permanent because the clean sector will never be so much more productive as to make demand for dirty intermediate production negligible. The learning subsidy, although it is decreasing, must last until no more learning can occur because the maximum productivity is reached (see Eq. (12)). In our numerical implementation, the convergence of the clean technology towards its maximum productivity is slow, so that the subsidy stays in place for the time horizon considered.\(^3\)

\(^3\) This is not to say that the optimal policy response must be permanent under any conceivable parameterisation of the model. An exception is for example the case of a very high regeneration rate, so that atmospheric quality can reach its pre-industrial level very quickly: no carbon tax is needed from then on. But this case is not relevant for physical reality as the regeneration of the atmosphere and associated natural systems in the carbon cycle will in physical reality not occur in the next centuries (IPCC, 2013). Similarly, as an addition to the model presented, one could include rising extraction costs for fossil resources into the model that would slow down productivity in the dirty sector so much as to make it uncompetitive with the clean sector. This scenario is however unlikely due to the enormous and easily accessible amount of cheap coal, which needs to be left underground for any serious efforts to limit global warming (see Allen et al., 2009; Edenhofer et al., 2011; IPCC, 2014b).
3.3. The size of the carbon lock-in as additional intervention: second-best policy with a single instrument

In a second-best policy scenario the government has only a single instrument available to maximize welfare. Even in this case it can significantly improve the market outcome. This second-best intervention requires that the single instrument is set significantly higher compared to the optimal intervention. Results are close to the social optimum: numerical solutions show that for the different cases of elasticities the second-best optimum does not produce losses greater than 0.2% BGE (for the case of only a carbon tax, the losses for only a subsidy being an order of magnitude lower), even if the first-best production and consumption paths differ markedly. This is an unsurprising result: both the climate and the learning externality impact the distribution of inputs to the dirty and clean sector. So one instrument set significantly higher than in the first-best optimum can correct much of the second externality.

This result is illustrated for the case that the carbon tax is the single instrument available to the government: Fig. 4 displays the first-best (= Pigouvian) carbon tax \( \tau_{1,1} \) compared to the single instrument carbon tax \( \tau_{1,1}' \) for the three cases of the substitution possibility.

Again, if substitution possibilities are poor, socially optimal mitigation is more difficult to achieve by policy intervention: the lower the substitution possibilities, the higher the carbon tax thus has to be set when used to overcome the lock-in even beyond simply correcting the climate externality.

This finding completes our thesis that substitution possibilities play an ambivalent role: while in Subsection 3.1 it was shown that in the unregulated outcome good substitution possibilities cause the highest welfare losses, the numerical results of Subsections 3.2 and 3.3 demonstrate that poor substitution possibilities require the highest policy intervention.

![Graphs showing the optimal policy mix required to reproduce the social optimum.](image)

Fig. 2. Comparison of the (unregulated) market and the social planner solution: the time paths of global warming and the share of the clean and the dirty sector for the different cases of the substitution elasticity.

![Graphs showing the optimal policy mix required to reproduce the social optimum.](image)

Fig. 3. The optimal policy mix required to reproduce the social optimum. Prices are given in % of a reference case, the value at \( t = 100 \) for \( \epsilon = 3 \).

3.3. The size of the carbon lock-in as additional intervention: second-best policy with a single instrument

In a second-best policy scenario the government has only a single instrument available to maximize welfare. Even in this case it can significantly improve the market outcome. This second-best intervention requires that the single instrument is set significantly higher compared to the optimal intervention. Results are close to the social optimum: numerical solutions show that for the different cases of elasticities the second-best optimum does not produce losses greater than 0.2% BGE (for the case of only a carbon tax, the losses for only a subsidy being an order of magnitude lower), even if the first-best production and consumption paths differ markedly. This is an unsurprising result: both the climate and the learning externality impact the distribution of inputs to the dirty and clean sector. So one instrument set significantly higher than in the first-best optimum can correct much of the second externality.

This result is illustrated for the case that the carbon tax is the single instrument available to the government: Fig. 4 displays the first-best (= Pigouvian) carbon tax \( \tau_{1,1} \) compared to the single instrument carbon tax \( \tau_{1,1}' \) for the three cases of the substitution possibility.

Again, if substitution possibilities are poor, socially optimal mitigation is more difficult to achieve by policy intervention: the lower the substitution possibilities, the higher the carbon tax thus has to be set when used to overcome the lock-in even beyond simply correcting the climate externality.

This finding completes our thesis that substitution possibilities play an ambivalent role: while in Subsection 3.1 it was shown that in the unregulated outcome good substitution possibilities cause the highest welfare losses, the numerical results of Subsections 3.2 and 3.3 demonstrate that poor substitution possibilities require the highest policy intervention.
3.4. The impact of a high learning curve on the transition

This subsection provides an elementary consideration about the dependence of the structural change on the learning curve. "Transition time" denotes the time taken in the social planner solution to reach a share of 80% of the clean sector from at least a 20% share of that sector. The transition time is calculated for two different learning curves for the clean technology. Besides the standard parameterisation of $\gamma = 0.27$ a low value of $\gamma = 0.2$ is considered that results in a higher learning curve (see Fig. C.6 in Appendix C).

Table 1 presents the values of the transition time for two different learning curves and three cases of the substitution elasticity. This measure characterizes the transition from the fossil-fuel based to the low-carbon economy as a gradual adjustment or an immediate switch: the higher the substitution elasticity, the shorter the transition time.

By comparison, the numerical solution to AABH’s model contains a rather abrupt switch (see Fig. 1 D of AABH) due to their knife-edge condition for innovation to happen in both sectors (see Lemma 1 and Fig. 1B of AABH): in that model the transition time is 15 years for $\epsilon = 10$ and 70 years for $\epsilon = 3$ for the discount value of $\rho = 0.001$ (per year). Although the two conceptions of technological progress are very different, our analysis indicates that the outcome of ABBH’s directed technical change corresponds to a learning curve of sudden extremely high competitiveness. Such an outcome is implausible due to path-dependencies in technological development, as discussed in Subsection 2.1.3.

3.5. Implications of a two degree target

Due to high uncertainties about economic damages and losses of human lives, standard cost-benefit-analysis is of limited normative cogency for evaluating policy responses to climate change. A more promising normative approach will seek to evaluate pathways of decarbonisation taking a guardrail on climate damages as given. Limiting the most severe impacts from climate change requires keeping global mean temperature below 2 °C (IPCC, 2014a; Lenton et al., 2008). This two degree target has become the focus of many political efforts to limit global warming and economic studies have demonstrated its feasibility (IPCC, 2014b).

Under the unconstrained cost-benefit analysis of climate damages, the socially optimal amount of global warming is significantly above 2 °C for low values of the substitution elasticity (see Fig. 2) in our model: for $\epsilon = 3$ it is 2.9 °C at $t = 175$. For this case we compute the additional policy intervention necessary to comply with a two degree target. We find that the carbon tax for the two degree target is significantly – eventually about ten times – higher than the carbon tax from the first-best optimum of an unconstrained cost-benefit analysis (see Fig. 5). Low substitution possibilities hence make ambitious mitigation very expensive and difficult to implement politically as they require aggressive carbon pricing towards the end of the decarbonisation. This again motivates to treat substitution possibilities as non-constant over time and potentially subject to further policy intervention.

3.6. Comparison to the findings of AABH

AABH argue that a high substitution elasticity between the carbon-intensive and the carbon-free sector of the economy facilitates the structural change from a carbon-intensive to a low-carbon economy as it requires an immediate, but less comprehensive and merely temporary policy intervention. We confirm this result with our model, but highlight that a high substitution elasticity also creates greater risks of a welfare loss from a lock-in. We disagree with AABH that an immediate, but temporary policy intervention is optimal: a permanent intervention is required if a more empirically plausible conception of advancing renewables is assumed.4 This is immediate from comparing Fig. 1A and C of AABH with Fig. 3 above. Our study can thus be seen as indicating that ABBH’s policy advice for fostering low-carbon structural change is limited in scope: it depends on a particular calibration of technological progress in the framework of directed technical change, the normative assumption of an unconstrained cost-benefit-analysis, the restriction to finding the optimal policy response and an elasticity of substitution between sectors that is constant over time. The lesson for

---

4 Another difference between AABH and the present model is that only in the former the unregulated outcome leads to an “environmental disaster” (AABH p. 141). Although this could in principle happen in the present model, it does not for the entire parameter range examined. This difference is once more due to the different conceptions of technological progress: in the present model, the fact that clean and dirty consumption are imperfect substitutes always generates some demand for clean production, which causes learning effects of this sector (and which is not the case in AABH). These learning effects lead to a greater competitiveness of clean over dirty production before global warming reaches the scale of an environmental disaster for any scenario examined in this study.
Appendix A. First-order conditions of decentralized agents

A.1. Household

The Lagrangian corresponding to the household’s maximization problem is:

\[
\mathcal{L}(C_0, ..., C_T, K_0, ..., K_T, \mu_0, ..., \mu_T) = \sum_{t=0}^{T} \left( U(S_t, C_t) \right) \frac{1}{(1+\rho)^t} + \mu_t(r_t K_t + w_t L_t + r_t C_t - K_{t+1} + (1-\delta)K_t). \tag{A.1}
\]

The optimal choice for the household is thus characterized by the following first-order conditions:

\[
\frac{\partial \mathcal{L}}{\partial C_t} = \frac{\partial U}{\partial C_t} \left( \frac{1}{(1+\rho)^t} - \mu_t \right) = 0 \tag{A.2}
\]

and

\[
\frac{\partial \mathcal{L}}{\partial K_t} = \mu_t (r_t + (1-\delta)) - \mu_{t-1} = 0. \tag{A.3}
\]

A.2. Final-good producer

The usual equilibrium price conditions apply including the carbon tax:

\[
p_1 + \tau_1 = \frac{\partial Y}{\partial Y_1}, \tag{A.4}
\]

\[
p_2 = \frac{\partial Y}{\partial Y_2}. \tag{A.5}
\]

A.3. Dirty firm

The usual static equilibrium conditions for the interest rate and the wage apply:

\[
r = p_1 \frac{\partial Y_1}{\partial K_1}, \tag{A.6}
\]

\[
w = p_1 \frac{\partial Y_1}{\partial L_1}. \tag{A.7}
\]

A.4. Clean firm

The standard static equilibrium conditions apply including the learning subsidy:

\[
r = (p_2 + \tau_2) \frac{\partial Y_2}{\partial K_2} \tag{A.8}
\]

\[
w = (p_2 + \tau_2) \frac{\partial Y_2}{\partial L_2}. \tag{A.9}
\]

Prices are then determined by the general equilibrium, that is by inserting Eqs. \((A.4)-(A.9)\) into Eq. \((3)\) and solving the optimality conditions of the household subject to Eq. \((3)\).
Appendix B. Optimisation problem of the social planner

The social planner problem given by Eqs. (4)–(12), (17)–(22) and the macroeconomic budget constraint \( Y_t = C_t + \ell_t \) can be simplified to the following system of equations:

\[
\begin{align}
\max_{C_t, K_t, L_t, \ell_t} & \sum_{t=0}^{T} U(C_t, \phi(S_t)) \frac{1}{(1 + \rho)^t} \\
0 & = -S_{t+1} - \xi Y_{1,t} + (1 + \xi) S_t \\
0 & = -K_{t+1} + Y(Y_{1,t}, Y_{2,t}) - C_t + (1 - \delta) K_t \\
0 & = -Y_{1,t} + F_1(K_t, L_t) \\
0 & = -Y_{2,t} + F_2(g(H_t), K_t - K_{1,t}, L - L_{1,t}) \\
0 & = -H_{t+1} + (Y_{2,t} - Y_{2,t-1}) + H_t,
\end{align}
\]

Here, we have abbreviated

\[
Y(Y_{1,t}, Y_{2,t}) = A_t \left( \frac{1}{Y_{1,t}^{1/\gamma}} + \frac{1}{Y_{2,t}^{1/\gamma}} \right)^{\gamma/(\gamma-1)}
\]

and

\[
g(H_t) = A_{2,t} = -\frac{\beta}{1 + \left( \frac{\mu}{H_t} \right)^\gamma}.
\]

The Lagrangian corresponding to the social planner's maximisation problem then is:

\[
L(C_t, K_t, S_t, L_t, \ell_t, Y_{1,t}, Y_{2,t}, H_t, \lambda_t, \mu_t, \psi_t, \kappa_t, \ldots, C_T, K_T, S_T, L_T, Y_{1,T}, Y_{2,T}, H_T, \lambda_T, \mu_T, \psi_T, \kappa_T) = \\
\sum_{t=0}^{T} U(C_t, \phi(S_t)) \frac{1}{(1 + \rho)^t} + \lambda_t (-S_{t+1} - \xi Y_{1,t} + (1 + \xi) S_t) + \\
\mu_t (-K_{t+1} + Y(Y_{1,t}, Y_{2,t}) - C_t + (1 - \delta) K_t) + \\
\nu_t (Y_{1,t} - F_1(K_t, L_t)) + \psi_t (-Y_{2,t} + F_2(g(H_t), K_t - K_{1,t}, L - L_{1,t})) + \\
\kappa_t (-H_{t+1} + (Y_{2,t} - Y_{2,t-1}) + H_t).
\]

The social optimum is thus characterised by the following conditions:

\[
\frac{\partial L}{\partial C_t} = \frac{\partial U}{\partial C_t} \frac{1}{(1 + \rho)^t} - \mu_t = 0
\]

\[
\frac{\partial L}{\partial S_t} = \frac{\partial U}{\partial S_t} \frac{1}{(1 + \rho)^t} - \lambda_t - 1 + \lambda_t (1 - \xi) = 0
\]

\[
\frac{\partial L}{\partial K_t} = -\mu_{t-1} + \mu_t (1 - \delta) + \psi_t \frac{\partial F_2}{\partial K_t} = 0
\]

\[
\frac{\partial L}{\partial Y_{1,t}} = -\xi \lambda_t + \mu_t \frac{\partial Y}{\partial Y_{1,t}} - \nu_t = 0
\]

\[
\frac{\partial L}{\partial Y_{2,t}} = \mu_t \frac{\partial Y}{\partial Y_{2,t}} - \psi_t + \kappa_t - \kappa_{t-1} = 0
\]

\[
\frac{\partial L}{\partial H_t} = \nu_t \frac{\partial F_1}{\partial H_t} - \psi_t \frac{\partial F_2}{\partial H_t} = 0
\]

\[
\frac{\partial L}{\partial \ell_t} = \nu_t \frac{\partial F_1}{\partial \ell_t} - \psi_t \frac{\partial F_2}{\partial \ell_t} = 0
\]

Appendix C. Parameter choices for numerical solution

Tables C.2 and C.3 detail all parameters for simulation. Since AABH do not state precisely their employed values for the emission intensity \( \xi \) and regeneration rate \( \gamma \) of the atmosphere, we compute our own emission intensity and regeneration rate with the values for current CO2 increase and dissipation given in Rezai et al. (2012). Fig. C.6 illustrates the role of the learning parameter \( \gamma \) for generating a higher learning curve as used for the analysis in Subsection 3.4.

Table C.2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta )</td>
<td>Intertemporal elasticity of substitution</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>Elasticity of substitution between clean and dirty sector</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Discount rate</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Factor intensity in production</td>
</tr>
<tr>
<td>( \xi )</td>
<td>Pre-industrial CO2-concentration: 280 ppm</td>
</tr>
<tr>
<td>( \Delta_{0} )</td>
<td>Current CO2-concentration: 389 ppm</td>
</tr>
<tr>
<td>( \lambda_1 )</td>
<td>Current CO2-concentration: 889 ppm</td>
</tr>
</tbody>
</table>

Table C.3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta )</td>
<td>Depreciation of capital</td>
</tr>
<tr>
<td>( g_s )</td>
<td>General productivity growth</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Maximum productivity</td>
</tr>
<tr>
<td>( \omega )</td>
<td>Scaling parameter</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Curvature of learning curve</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Emission intensity</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>Regeneration capacity of atmosphere</td>
</tr>
<tr>
<td>( K(0) )</td>
<td>Initial value of capital stock</td>
</tr>
<tr>
<td>( H(0) )</td>
<td>Initial value of knowledge</td>
</tr>
<tr>
<td>( L(t) )</td>
<td>Normalized size of labour force over all time periods</td>
</tr>
</tbody>
</table>

Fig. C.6. Two learning curves differing by the learning parameter \( \gamma \) for the clean technology.
Appendix D. Additional figures representing the lock-in

![Figures showing consumption and damages](image1)

Fig. D.7. Comparison of the (unregulated) market and social planner solution: the time paths of consumption (in monetary units, given initial value for capital) and damages (share of remaining consumption relative to pre-industrial environmental quality).

![Figures showing investment and capital stocks](image2)

Fig. D.8. Comparison of the (unregulated) market and social planner solution: the time paths of total investment and capital stocks in both sectors (in monetary units, given initial value for capital).

Appendix E. Sensitivity analysis

We first check for robustness of the main results across variations of the crucial parameter for this study, the substitution elasticity between the clean and the dirty sector \( \varepsilon \). We then provide a general sensitivity analysis and find that our main results are robust for all parameter variations displayed in Table E.5. Results of extensive further sensitivity checks are available as Supplementary Information online.

E.1. The substitution elasticity between clean and dirty production

Figs. E.9 and E.10 present the differences in the time-paths of the socially optimal and the unregulated market outcome for further values of \( \varepsilon \). Moreover, Table E.4 quantifies the welfare loss through the lock-in for a broader range of values. The analysis confirms that the “low” and “high” case considered in the main part of the article capture all relevant differences. For the limiting cases, one observes the following outcome: as \( \varepsilon \) approaches 0, the sectoral share
of clean and dirty sectors approach 0.5 in both decentralized version and socially optimal allocation, in accordance with the limiting case of a Leontief production function. For $\varepsilon > 11$ and the case of the unregulated market outcome, no transformation towards the clean sector occurs within the considered time frame anymore.

E.2. Parameter variation

We now provide a sensitivity analysis for all parameters (see Appendix C) for this study. We check the robustness of our main results by giving the welfare loss in the unregulated outcome across parameter values in Table E.5. Moreover, we check whether, for the parameter combinations tested, a change in the time-paths of the socially optimal and the unregulated market outcome occurs (in comparison to Fig. 2, more on this in the Supplementary Information). We find that for the

<table>
<thead>
<tr>
<th>Value of $\varepsilon$</th>
<th>Welfare loss in % BGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.74</td>
</tr>
<tr>
<td>3</td>
<td>1.14</td>
</tr>
<tr>
<td>4</td>
<td>1.59</td>
</tr>
<tr>
<td>5</td>
<td>2.07</td>
</tr>
<tr>
<td>6</td>
<td>2.53</td>
</tr>
<tr>
<td>7</td>
<td>2.89</td>
</tr>
<tr>
<td>8</td>
<td>3.13</td>
</tr>
<tr>
<td>9</td>
<td>3.36</td>
</tr>
<tr>
<td>10</td>
<td>3.66</td>
</tr>
<tr>
<td>11</td>
<td>3.90</td>
</tr>
<tr>
<td>12</td>
<td>3.91</td>
</tr>
</tbody>
</table>

Fig. E.9: Sensitivity of the comparison of the (unregulated) market and the social planner solution: the time paths of global warming and the share of the clean and the dirty sector for the further cases of the substitution elasticity.

Fig. E.10: Sensitivity of the comparison of the (unregulated) market and the social planner solution: the time paths of global warming and the share of the clean and the dirty sector for the further cases of the substitution elasticity.

Table E.4

The lock-in of the unregulated market outcome for further substitution elasticities: comparison to the socially optimal outcome in terms of welfare given in % BGE.
ranges shown, results do not change qualitatively, ensuring that the policy implications also do not change. We subsequently explain the changes in welfare losses in the unregulated outcome with respect to varying parameters displayed in Table E.5. Welfare losses as functions of a single parameter are either monotonically increasing, monotonically decreasing or inverted U-shaped. They remain in the range of 1–3% BGE.

For the elasticity of intertemporal substitution $\eta$, welfare losses are monotonically increasing. A higher $\eta$ means that an unequal distribution of consumption over time lowers welfare. This is true in this model because consumption losses due to the lock-in occur early on, while the economy is growing, so that a higher $\eta$ produces high welfare losses. A higher rate of pure time preference $\rho$ lowers welfare losses, as expected. Later consumption losses are discounted more and also much less investment and growth occurs, so that damages are lower. A higher capital intensity in production $\theta$ leads to results similar to a lower substitution elasticity: a more capital-intensive economy has higher socially optimal levels of warming and the lock-in occurs earlier and is smaller, which leads to decreasing welfare losses.

With a higher disaster temperature parameter, disaster levels translate into lower utility losses. Thus welfare losses are lower for a higher disaster temperature parameter. Similarly, with higher $\lambda$, damages have a greater weight in the utility function, so welfare losses increase.

Welfare losses are highest for the standard depreciation rate $\delta$, they are slightly lower for low rates and much lower for high depreciation rates. For low depreciation rates, the delay of the transition for the decentralized case is smaller, leading to slightly reduced welfare losses. For high depreciation rates, although the delay is greater, welfare losses are lower due to much lower climate damages, because the size of the economy is much smaller.

Lower values for general productivity growth $g_s$ lead to lower damages as the economy is smaller, and hence to lower welfare losses. Higher values also lead to lower welfare losses because much less of a lock-in occurs. This is because if the economy is much more productive, learning happens much quicker in the decentralized equilibrium by comparison to the socially optimal case.

The key parameters governing the learning of the clean sector $\beta$ and $\gamma$ both yield an inverted U-shape curve of welfare losses. For $\beta$, the maximum productivity of the learning technology, high rates mean that learning happens fast both in the social planner and the decentralized version, so there is little lock-in and hence little welfare losses. On the other hand, low rates mean that the transition happens slowly for the social planner solution, so that welfare losses are also lower compared to the standard case. The lock-in is hence greatest for a medium maximum productivity of the learning technology. The same is true for varying $\gamma$, the curvature of the learning curve. Low values of $\gamma$ lead to high learning rates, while high values mean that learning happens slowly (see Appendix C). The results for varying $\gamma$ are thus similar to $\beta$, but reversed. A lower scaling parameter $\omega$ makes the social planner solution switch more rapidly than the decentralized version, whence welfare losses are higher.

As the behaviour with respect to varying depreciation, productivity growth and learning seems most important and relevant to the main conclusions of the study, the Supplementary Information (available upon request) contains further Figures (akin to Fig. 2 above) illustrating the behaviour described here.

Finally, the higher the emission intensity $\xi$ and the lower the regeneration rate $\zeta$, the greater are the welfare losses: although socially optimal damages are somewhat higher in these cases, the unregulated damages are more than proportionally higher.

Appendix F. Supplementary data

Supplementary data to this article can be found online at http://dx.doi.org/10.1016/j.econmod.2015.06.002.

References


Table E.5

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Low value</th>
<th>Std. value</th>
<th>High value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity of intertemp. subst. $\eta$</td>
<td>1.1</td>
<td>1.5</td>
<td>2</td>
</tr>
<tr>
<td>Welfare loss in % BGE</td>
<td>1.22</td>
<td>2.02</td>
<td>2.53</td>
</tr>
<tr>
<td>Pure rate of time preference $\rho$</td>
<td>0.01</td>
<td>0.03</td>
<td>0.00501</td>
</tr>
<tr>
<td>Welfare loss in % BGE</td>
<td>2.58</td>
<td>2.56</td>
<td>2.53</td>
</tr>
<tr>
<td>Factor intensity in intermediate production</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>(capital share) $\theta$</td>
<td>2.73</td>
<td>2.65</td>
<td>2.53</td>
</tr>
<tr>
<td>Disaster temperature $\Delta_{\text{dis}}$</td>
<td>6</td>
<td>6.5</td>
<td>6.9</td>
</tr>
<tr>
<td>Corresponding initial value of environmental quality $S_0$</td>
<td>731</td>
<td>868.2</td>
<td>989.9</td>
</tr>
<tr>
<td>Welfare loss in % BGE</td>
<td>2.76</td>
<td>2.60</td>
<td>2.53</td>
</tr>
<tr>
<td>Damage scale parameter $\lambda$</td>
<td>0.1</td>
<td>0.3</td>
<td>0.3492</td>
</tr>
<tr>
<td>Welfare loss in % BGE</td>
<td>2.19</td>
<td>2.45</td>
<td>2.53</td>
</tr>
<tr>
<td>Depreciation of capital $\delta$</td>
<td>0.05</td>
<td>0.1</td>
<td>0.141</td>
</tr>
<tr>
<td>Welfare loss in % BGE</td>
<td>2.49</td>
<td>2.52</td>
<td>2.53</td>
</tr>
<tr>
<td>General productivity growth $g_s$</td>
<td>0.021</td>
<td>0.035</td>
<td>0.049</td>
</tr>
<tr>
<td>Welfare loss in % BGE</td>
<td>2.22</td>
<td>2.50</td>
<td>2.53</td>
</tr>
<tr>
<td>Maximum clean productivity $\beta$</td>
<td>6.5</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>Welfare loss in % BGE</td>
<td>1.08</td>
<td>1.60</td>
<td>2.53</td>
</tr>
<tr>
<td>Scaling Parameter $\omega$</td>
<td>200</td>
<td>250</td>
<td>300</td>
</tr>
<tr>
<td>Welfare loss in % BGE</td>
<td>2.65</td>
<td>2.63</td>
<td>2.53</td>
</tr>
<tr>
<td>Curvature of learning curve $\gamma$</td>
<td>0.2</td>
<td>0.24</td>
<td>0.27</td>
</tr>
<tr>
<td>Welfare loss in % BGE</td>
<td>1.04</td>
<td>1.99</td>
<td>2.53</td>
</tr>
<tr>
<td>Emission intensity $\xi$</td>
<td>1</td>
<td>1.4</td>
<td>1.7</td>
</tr>
<tr>
<td>Welfare loss in % BGE</td>
<td>2.27</td>
<td>2.41</td>
<td>2.53</td>
</tr>
<tr>
<td>Regeneration rate $\zeta$</td>
<td>0.0005</td>
<td>0.001</td>
<td>0.00137</td>
</tr>
<tr>
<td>Welfare loss in % BGE</td>
<td>2.55</td>
<td>2.54</td>
<td>2.53</td>
</tr>
</tbody>
</table>

This parameter must be varied with the resulting initial level of environmental quality in order to leave current CO2 concentrations fixed, see Subsection 2.1.4.


Linus Mattauch is a researcher at the Mercator Research Institute on Global Commons and Climate Change (MCC) in Berlin. Besides climate change economics, his research interests are the interaction of climate policy with public finance and welfare implications of the behavioural sciences.

Felix Creutzig leads the working group Land Use, Infrastructures and Transport at the Mercator Research Institute of the MCC. He is lead author of the IPCC’s Fifth Assessment Report and lead analyst of the Global Energy Assessment. His research focuses on: conceptualising and assessing opportunities for mitigating GHG emissions of cities, building models of sustainable urban form, and land use-mediated uncertainty in integrated assessments. He previously was a postdoctoral fellow at the Energy and Resources Group at the University of California and the Princeton Environmental Institute.

Ottmar Edenhofer is director of the MCC, professor of the economics of climate change at the Technical University of Berlin and co-chair of the Working Group III of the Intergovernmental Panel on Climate Change (IPCC). He is deputy director and chief economist at the Potsdam Institute for Climate Impact Research and is currently leading Research Domain III: Sustainable Solutions there. His research explores the impact of induced technological change on mitigation costs and mitigation strategies, as well as the design of instruments for climate and energy policy.